M1220-2 **Final Review Questions** **Spring 2005**

Chapter 7

1. Functions and their inverses: How was the natural log of x, the function $f(x) = \ln x$ defined? What is its domain and range, and what does its graph look like. Why is it differentiable? Why does it have an inverse function and what is its inverse function? What is the relationship between the derivative of a function and the derivative of the inverse function when the inverse exists? What is logarithmic differentiation and why is it useful?

2. How can you describe the number e in terms of limits and geometrically in terms of a graph? What is the solution to the differential equation $\frac{dy}{dx} = ky$, for k a constant? What is the equation for populations growth and how can you use it in applications? What is doubling time and half life? Can we evaluate limits that involve e?

3. How does one differentiate and integrate functions that involve: $e^x, \ln x$, trig functions and the inverse trig functions?

Chapter 8

4. Integration Techniques: What are the two main techniques one should always remember because they occur most often? How do we use the half angle and double angle formulas to evaluate special trig integrals such as $\int \cos^n(x)dx, \int \sin^n(x)dx, \int \cos^n(x)\cos^m(x)dx$? What is integration by partial fractions?

Chapter 9

5. What is L'Hopital's Rule? What kinds of limit questions can be reduced to a form where l'Hopital's Rule applies? When is it better to find $\lim_{x\to+\infty}[\ln f(x)]$ and in that case what do we do with the answer?

6. What is an improper integral? Are there different kinds? How are these integrals defined? What can you say about the convergence/divergence of $\int_1^{+\infty} \frac{`1}{x^p}dx$, or $\int_0^1 \frac{`1}{x^p}dx$, for different values of p?

Chapter 10

7. What is a sequence? What is an infinite series? What is the difference between a sequence and a series? How can we tell whether a sequence will have a limit or not? What does it mean to say that an infinite series converges? What tests can we use to tell if a series converges or diverges? What are the special infinite series we use as guideposts. Can we give examples of these special series that converge/diverge? What is a power series in x, and

a power series in $(x-a)$? What is the convergence set of a power series? what is the radius of convergence. What is the Taylor series expanded about a for a function f that has derivatives of all orders at $x=a$? What if the remainder term for the nth Taylor polynomial? What are the special Taylor or Maclaurin series we should know?

Chapter 11

8. What is the principle behind Newton's Method for finding roots of equations and how do we use this method? What is the principle behind the Trapezoid Rule and we do we use this rule?

Chapter 12

9. What are rectangular coordinates, polar coordinates and their properties? How do we change back and forth between rectangular to polar coordinates? How can we determine if a second degree equation in x,y represents a parabola, an ellipse, or a hyperbola? What are the special curves in polar coordinates? How do we find area bounded by these curves in polar coordinates?

!0. True or false:

a) Suppose that $g(x) = \int_0^x (3t^2+2)dt$. Then $g(1) = 3$ and $(g^{-1})'(3) = 1/5$.

b. If $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x)$, then $\lim_{x\to 0} \frac{f(x)}{g(x)} = 1$.

c. If $p > 0$ and $\int_1^{+\infty} \frac{1}{x^p} dx$ converges, then $\int_0^1 \frac{1}{x^p} dx$ may also converge.

d. If $\lim_{n\to+\infty} |a_n| = 0$, then $\lim_{n\to+\infty} a_n = 0$.

e. If $\lim_{n\to+\infty} |a_n| = 2$, then $\lim_{n\to+\infty} a_n = 2$.

f. If $p > 0$ and $\int_1^{+\infty} \frac{1}{x^p} dx$ diverges, then $\int_0^1 \frac{1}{x^p} dx$ must converge.

g. If a sequence alternates between positive and negative numbers, then the sequence has a limit.

h. If an infinite series converges, then the the terms in the series must approach zero as n approaches infinity.

i. If an infinite series diverges, then the terms in the series do not approach zero.

j. If the series $\sum |a_n|$ converges, then the series $\sum a_n$ converges.

k. If $\sum a_n, \sum b_n$ both diverge, then $\sum(a_n + b_n)$ will diverge. l. If $\sum a_n, \sum b_n$ both diverge, then $\sum(a_n b_n)$ will diverge. m. If $\sum a_n, \sum b_n$ both converge, then $\sum(a_n + b_n)$ will converge.

n. If $\lim_{n\to+\infty} a_n = 0$, then the infinite series $\sum a_n$ converges.

o. In the Cartesian coordinate system each point in the plane is represented by a unique ordered pair of real numbers.

p. In the Polar coordinate system each point in the plane is represented by a unique ordered pair of real numbers.

q. If f is an even function and $\int_0^{+\infty} f(x)dx$ converges, then $\int_{-\infty}^{+\infty} f(x)dx$ converges.

r. If $0 \leq f(x) \leq e^{-x}, x \geq 0$, then $\int_0^{+\infty} f(x)dx$converges.

s. $\sum_1^{+\infty}(1/n)^n$ converges and has a sum between 1 and 2.

t. If a series diverges then its sequence of partial sums is unbounded.

u. If a positive term series diverges, then its sequence of partial sums is unbounded.

v. If a power series$\sum a_n x^n$ converges at $x = 2$, then the series converges for $x = -1$.

w. If a power series$\sum a_n x^n$ converges at $x = 2$, then the series converges for $x = -2$.

x. If $\sum_0^{+\infty} a_n x^n$ is the Maclaurin series for the function f and it converges to f for all $x, |x| < 3$, then the Maclaurin series for f'(x) $= \sum_1^{+\infty} n a_n x^{n-1}$ and this series converges to $f'(x)$, whenever $|x| < 3$.

y. The Taylor polynomial of order n based at a for the function f(x) is unique. That is, f has only one such polynomial.

z. If f is continuous on [a, b] and $f(a)f(b) < 0$, then f has a root in (a,b).

Sample Test Problems from Text:
p365: 5, 15,17,27,37,38 p399: 3,9,27,35,37 p425: 1,5,7,9,15,19,21,23,31 p475: 3,7,13,15,17,25,27,33,34,35,37,46 p512: 2a,5 p556: 3,5,25,29